Joint Centre for Mesoscale Meteorology, Reading, UK

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A. J. Thorpe
I. A. Renfrew
C. H. Bishop

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Met Office Joint Centre for Mesoscale Meteorology Department of Meteorology
University of Reading PO Box 243 Reading RG6 6BB United Kingdom
Tel: +44 (0)118 931 8425 Fax: +44 (0)118 931 8791
www.metoffice.com

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The Role of the Environmental Flow in the Development of Secondary Frontal Cyclones

By Ian A Renfrew[1*], Alan J Thorpe[2] and Craig H Bishop[3]

[1]*Dept. of Physics, University of Toronto, Canada*

[2]*Dept. of Meteorology, University of Reading, UK*

[3]*Dept. of Meteorology, Pennsylvania State University, USA*

[*]Corresponding author: Department of Physics, University of Toronto, 60 St George Street, Toronto, Ontario M.S. 1A7, Canada.

SUMMARY

The impact of the environmental flow on the development of secondary frontal cyclones is investigated. Several case studies are examined as examples of secondary frontal cyclone events observed in the North Atlantic - Western Europe sector. A simple measure of growth is defined to chart their development. The vorticity attribution technique of Bishop (1996I&II) is utilised to calculate the action of the large scale (environmental) flow on the fronts. In particular the environmental along-front stretching - shown to be important in theoretical models of frontal instabilities - is calculated. The role of the environmental deformation appears to be crucial: as part of a baroclinic lifecycle, stretching deformation acts to build up a front but suppress along-front waves; if the stretching drops off, barotropic instabilities may then break out. Diagnostics are examined to try to ascertain the growth mechanisms at work in each frontal cyclone case. A range of values for the commonly prescribed deformation frontogenesis and shearing frontogenesis parameters are calculated.

1. INTRODUCTION

The downstream tail of the North Atlantic synoptic stormtrack appears to be the area most energetic in terms of sub-synoptic scale cyclonic activity (Ayrault *et al.* 1995). One subset of these subsynoptic cyclones is that of secondary frontal cyclones, defined as waves on the pre-existing frontal zones of primary cyclonic systems. Such secondary frontal cyclones are an important and common local forecasting problem in western Europe, especially as they occasionally deepen explosively into full blown storms (eg. Hoskins and Berrisford 1988). Ordinary secondary frontal cyclones (ie. those that do not deepen explosively) are typically up to 1000km in scale, fast growing (e-folding times of 1 day) and short lived (lifetimes of 1-2 days). Figure 1 shows two such frontal cyclones marked on the trailing cold front of a synoptic scale low system south of Greenland. This is an interesting case as the two frontal cyclones appear similar in size and structure initially, but their evolutions are decidedly different. An explanation for this difference is put forward in §3.

Theoretical modeling studies have found instabilities on the above lengthscales and timescales if the frontal zone includes a low level extremum in potential vorticity (Joly and Thorpe 1990, Malardel *et al.* 1993, Polavarapu and Peltier 1993); or equivalently a boundary potential temperature anomaly (Schär and Davies 1990, Joly 1995). These situations are unstable to barotropic or internal baroclinic instabilities via the Charney-Stern (1962) instability criterion. The wave amplification can be thought of in terms of a phase locking and mutual amplification of edge waves on the edges of the potential vorticity strip or warm band (see for example Hoskins *et al.* 1985). However when the action of the environmental flow (ie. the baroclinic wave in which the front is embedded) is included, this is also found to be crucial to any frontal wave development. For example in Thorncroft and Hoskins (1990) the most unstable modes (about 1000km in scale) were torn apart when added to a baroclinic lifecycle experiment.

The two most important environmental flow mechanisms instrumental in forming and intensifying fronts are those of horizontal shearing and horizontal deformation.

Several recent studies have examined the impact of these two frontogenesis mechanisms on the development of along-front waves. Joly and Thorpe (1991) investigated the growth of disturbances on a horizontal shearing forced front. For moist frontogenesis there was the possibility of rapid frontal cyclogenesis dependent upon two factors: the rate of frontogenesis and the state of the parent baroclinic wave. The authors suggest that the direct action of horizontal shearing deformation does not affect frontal wave growth; but indirectly larger shearing frontogenesis leads to a more potentially unstable front.

Bishop and Thorpe (1994I&II, hereafter *BT94I&II*) specifically investigated barotropic edge wave growth on an idealized moist tropospheric front in an external deformation flow. It was found that wave amplification depended upon the magnitude of the potential vorticity (PV) of the strip and the scale of the disturbance relative to the strip width. While the deformation flow and induced ageostrophic convergence act (via latent heat release) to increase the PV of the strip, they also tend to flatten nascent frontal waves. Furthermore the deformation frontogenesis process changes the non-dimensional wavelength of the waves through time - pushing them out of the 90° phase shift required for maximum amplification. Hence there is a trade off between keeping the waves in phase for as long as possible, and keeping the waves in phase when the PV of the strip is large and waves will amplify more rapidly. A stretching deformation rate of greater than $0.6\times10^{-5}s^{-1}$ suppresses most wave growth at all scales; whereas a lower deformation rate of (for example) $0.2\times10^{-5}s^{-1}$ allows large amplification of frontal cyclone scale waves.

A highly idealized study by Dritschel *et al.* (1991) examined the role of external horizontal deformation and external horizontal shearing on edge wave growth. Of the two they found the suppressing action of the external deformation flow was the primary effect. The role of deformation on baroclinic instabilities is less clear cut, see Bishop (1993I&II).

It is clear from the above studies that both a potential for frontal instability and the evolving environmental flow are decisive in the development of any along-front waves. So for a proper analysis of observational data both the potential instability and the action of the environmental flow must be examined. The difficulty is that obtaining the relevant information from the hugely complicated observational data is fraught with problems: for

example the local structure of the front masks the action of the environmental flow on the front. To compare the real atmosphere to theoretical models one really needs a way of *partitioning* the observed flow into separate parts - for example a *frontal* part and an *environmental* (background) part. This would allow a comparison between the front and its environment in the real atmosphere, and the front and its environment in a theoretical model. In other words such a partition would allow an analogy between the potential instability and its environment, and the perturbation and basic state parts of theoretical models. The two separate parts could then be analysed individually and relevant diagnostics calculated.

One way to achieve this partition is using an *attribution technique*, where an anomaly is isolated and a particular flow field is associated or attributed to that anomaly in some coherent manner. Utilising such a technique in a practical way has been the subject of recent research (eg. Hoskins *et al.* 1985, Davis and Emanuel 1991, Davis 1992, Bishop and Thorpe 1994c). The technique used in this study is that of domain independent vorticity and divergence attribution developed by Bishop (1996I&II). This technique is summarised in section 2. Using domain independent attribution allows the accurate calculation of the external or environmental stretching deformation rate shown to be crucial to frontal wave development in theoretical studies. In section 3 the relationship between frontal cyclone growth and the environmental stretching deformation is investigated for a series of real case studies. Section 4 synthesises the results of all the case studies, the aim being to provide a quantitative relationship between stretching deformation and frontal cyclone growth. Section 5 concludes the paper.

2. ANALYSIS TECHNIQUES

(a) Domain Independent Attribution

A horizontal wind field on a periodic or global domain can be uniquely partitioned[1] into *irrotational* and *nondivergent* parts (Helmholtz Theorem, see Morse and Feshbach 1953 for example). So for a horizontal wind $\boldsymbol{u}$,

$$\boldsymbol{u} = -\nabla\times(\psi\boldsymbol{k}) + \nabla\chi = \boldsymbol{u}_\psi + \boldsymbol{u}_\chi \qquad (1)$$

with streamfunction ψ and velocity potential χ, where $\boldsymbol{k}$ is the unit vector in the vertical. So $\boldsymbol{u}_\psi$ is *nondivergent* and $\boldsymbol{u}_\chi$ is *irrotational*. Defining the vorticity as $\xi = \boldsymbol{k}.\nabla\times\boldsymbol{u}$ and the divergence as $\delta = \nabla.\boldsymbol{u}$, then combining these and (1) leads to the Poisson equations:

$$\nabla^2\psi = \xi \quad \textit{and} \quad \nabla^2\chi = \delta \qquad (2)$$

So the nondivergent part of the wind $\boldsymbol{u}_\psi$ is due to *vorticity* anomalies and the irrotational part of the wind $\boldsymbol{u}_\chi$ is due to *divergence* anomalies.

It is well known that such a unique partition is *not* possible on a finite domain, where the partition depends upon the boundary conditions imposed (eg. Lynch 1988, 1989). The approach of Bishop (1996I) bypasses this problem by solving the Poisson equations (2) using *free-space* Green's functions ie. where the boundary conditions are removed to infinity. This introduces a certain domain independence to the solutions. In 2 dimensions the Green's function is $ln(r)$, where r is the radial distance from the centre of the disc, so the induced winds are proportional to $1/r$. For discrete data in a finite domain,

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[1]Note here the *partitioning* problem is that of separating a given wind field into irrotational and nondivergent parts as described for example in Lynch (1988). The word partition is also used in a more general sense elsewhere to mean the division of a wind field in some way. Closely related is the *reconstruction* problem which concerns the calculation of a wind field from given vorticity and divergence distributions (with winds given at the domain boundaries).

a unique wind field is attributed to each element of vorticity and divergence by calculating the wind field due to a disc of uniform vorticity and divergence assigned to each grid point in the domain. The vorticity and divergence of the discs are calculated via sophisticated circulation and flux estimates using wind data from the neighbouring grid points. To obtain the total *rotational* flow $\boldsymbol{u}_\psi$ (the flow due to all the vorticity in the finite domain) the wind induced by all the vorticity grid discs is summed. To obtain the total *divergent* flow $\boldsymbol{u}_\chi$ the wind induced by all the divergence grid discs is summed. As the domain is finite there is also a remainder flow: the *harmonic* part of the wind $\boldsymbol{u}_\vartheta$ defined by

$$\boldsymbol{u}_\vartheta = \boldsymbol{u} - \boldsymbol{u}_\psi - \boldsymbol{u}_\chi \qquad (3)$$

The harmonic wind is both solenoidal and irrotational in the domain, and so can be represented as a streamfunction ($\boldsymbol{u}_\vartheta = -\nabla\times\vartheta\boldsymbol{k}$) or velocity potential ($\boldsymbol{u}_\vartheta = \nabla\vartheta$). It can be calculated independently by solving

$$\nabla^2\vartheta = 0 \quad \textit{in the interior,} \quad \textit{subject to } \frac{\partial\vartheta}{\partial n} = \boldsymbol{n}.\boldsymbol{u}_\vartheta \qquad (4)$$

where $\boldsymbol{n}.\boldsymbol{u}_\vartheta$ is calculated via (3) on the finite domain boundary. So the attribution technique allows a unique and unambiguous three component partition of the wind on a finite domain that is independent of the domain boundaries:

$$\boldsymbol{u}_{obs} = \boldsymbol{u}_\psi + \boldsymbol{u}_\chi + \boldsymbol{u}_\vartheta \qquad (5)$$

The rotational part of the wind is due to vorticity, the divergent part due to divergence, and the harmonic part due to vorticity and divergence outside the domain. Hence there is a clear relationship between a vorticity or divergence anomaly and the wind field induced by it.

The details of the adaptation of the technique into a discrete formulation suitable for limited areas of gridded wind data is described in Bishop (1996I). Note the

reconstruction of the wind field can be achieved to a high degree of accuracy: root-mean-square and maximum wind errors are typically $0.1ms^{-1}$ and $0.25ms^{-1}$ respectively.

(b) Frontal Flow and Environmental Flow

The attribution method allows the association of a particular wind field to each grid point element of vorticity and divergence, so one can partition (or divide) a wind field by partitioning the vorticity and divergence distributions. In other words one can specify a particular region of the domain, and examine the flow due to vorticity and divergence in that region only. Following the ideas of Bishop (1996II) one can "draw a box around" a cold front (Fig. 2) and define the *frontal* flow as the wind induced by vorticity and divergence anomalies inside the box, and the *external or environmental* flow as the wind induced by vorticity and divergence anomalies external to the frontal box (including anomalies outside the finite domain). Hence there is a division of the wind field into these two distinct parts:

$$\boldsymbol{u}_{obs} = \boldsymbol{u}_e + \boldsymbol{u}_i = \{\boldsymbol{u}_{e\psi} + \boldsymbol{u}_{e\chi} + \boldsymbol{u}_{\vartheta}\} + \{\boldsymbol{u}_{i\psi} + \boldsymbol{u}_{i\chi}\}, \qquad (6)$$

where $\boldsymbol{u}_e$ is the wind due to vorticity and divergence external to the frontal region (rotational + divergent + harmonic parts) and $\boldsymbol{u}_i$ is the wind due to vorticity and divergence inside the frontal region (rotational + divergent parts).

So to investigate the impact of the large scale flow on a front one would examine the environmental wind $\boldsymbol{u}_e$. This wind is due to ξ and δ anomalies distant from the front and so would be expected to be relatively smooth in the frontal region. It is this division into frontal and environmental flows that allows a comparison with theoretical model fronts and their division into perturbation parts and balanced basic states. The prescribed *frontal box* may be defined as any shape: it is simply defined by the grid points that are inside the box (frontal) and those that are outside (environmental). In this study the frontal box is defined to enclose the strip of positive vorticity that typically delineates a baroclinic frontal zone, with the box edges parallel to the strip. So for fronts and frontal waves the

box used is either a rectangle or a segment of an annulus, thus covering both straight and curved fronts. At subsequent times the position of the frontal box is moved so that approximately the same frontal area is enclosed.

To calculate the magnitude of the deformation flow shown to be crucial to modelled frontal wave development (Dritschel *et al.* 1991, *BT94II*), the along-front stretching due to the environmental flow is measured by $\partial v_e/\partial y_f$. Note there is a rotation of co-ordinates into a frontal co-ordinate system so (x_f, y_f) are the (across-front, along-front) directions respectively. The along-front direction is defined parallel to the frontal box sides, or as the arc through the centre of the frontal annulus. The across-front direction points towards the warm air for a cold front. Note for convenience the subscript *(f)* on the velocity is dropped and the subscript on the co-ordinate is retained.

As discussed in Bishop (1996II) if the stretching of the observed winds is used to estimate the deformation rate then serious errors may occur. Firstly the local structure of the front will cause along-front waves in the stretching pattern, and secondly a small variation in angle can cause a large change in stretching rate (due to the large shear associated with the front). Calculating stretching from the environmental winds dramatically reduces these errors, by `removing' the frontal shear. A sensitivity study in Renfrew (1995) [where the frontal box size and angle were varied by ±25% in width and ±3° in angle] found a variation of only 3% in $\partial v_e/\partial y_f$, from the prescribed deformation rate in an idealized front, and only $0.06\text{x}10^{-5}\text{s}^{-1}$ (10%) from the original box estimate in a real data case. This can be compared to variations of 15% and $0.25\text{x}10^{-5}\text{s}^{-1}$ respectively in calculating $\partial v_{obs}/\partial y_f$.

(c) A Measure of Frontal Wave Amplitude

A key issue is how to measure the size and growth of an observed frontal wave. Most analytical studies (eg. Dritschel *et al.* 1991, *BT94I&II*) use amplification of waveslope as a measure of growth, as it is proportional to the amount of nonlinear growth. Essentially the waveslope is the angle between an isoline of vorticity (for example) and its basic state

position; on a horizontal plane $\partial\xi'/\partial y_f$. Unfortunately defining and calculating waveslope in real data has a large error associated with it. To be measured the waveslope must be defined relative to some "undisturbed" frontal position, and in observed fronts this undisturbed position is a conjectural matter. Ideally the measure of growth should be relatively insensitive to the chosen along-front direction. However as waveslope is defined relative to this direction it is intrinsically directly dependent upon it.

To overcome this problem a new measure of along-front variation has been developed starting from a practical, observational perspective rather than a theoretical one. The new measure of growth is the *peak vorticity* minus the *maximum along-front average vorticity* of the strip:

$$W(\xi) = \xi_{peak} - \max\{\bar{\xi}^y\} \qquad (7)$$

where y is the along-front direction and the maxima are taken over some specified "frontal region" ie. the frontal box. We have called $W(\xi)$ the *vorticity wavyness*. The main advantages of $W(\xi)$ are that it is straightforward to calculate in observed flows, and does not depend directly on the along-front direction - only indirectly through the along-front average term. It appears from the real data sensitivity studies in Renfrew (1995) that $W(\xi)$ is relatively insensitive to small changes in the along-front direction - such changes give variations less than $0.06\times10^{-4}s^{-1}$ (8%) from the original box value. Vorticity wavyness is a simpler measure than waveslope, which means it should be less affected by small scale noise. However it is sensitive to model resolution (as are all vorticity measures) and to the length of the frontal region specified. Care needs to be taken that this vorticity strip length is defined consistently through the instability evolution - another reason for a "Lagrangian" approach to defining the frontal box.

In Renfrew (1995) instabilities of a strip of vorticity in a numerical barotropic vorticity equation model are investigated. The amplitudes of the instabilities are measured using both waveslope and vorticity wavyness, and these measures are shown to be comparable through the evolution of the frontal instability. This result provides some

justification for the use of $W(\xi)$ on real data and interpreting the results in the context of other analytical work.

3. CASE STUDIES

A series of frontal cyclone case studies was investigated and the results are outlined below. A broad range of events was chosen and so each case is quite individual. However a detailed description of each case is beyond the scope of this paper and instead the reader is referred to Renfrew (1995) or to a forthcoming paper by Renfrew (1996). The key goals in the investigation are:

- measuring the growth of the frontal wave.
- examining the environment of the front in suppressing or allowing frontal wave growth.
- suggesting probable growth mechanisms on the basis of calculated diagnostics.
- measuring key parameters usually prescribed in frontal theory ie. the deformation rate α, and the horizontal shearing frontogenesis parameter s^2 (cf Hoskins and Bretherton 1972).

Note the data used in this study and taken as observational truth are analyses from the UK Meteorological Office Unified Model (Limited Area Version) at 00GMT, 06GMT, 12GMT and 18GMT - which is also approximately local time in each case (ie. 00h=00GMT).

(a) Case of 12th-13th November 1994

Figure 1 shows the sea-level pressure analysis for 12h on the 12th November 1994, where two frontal cyclones are analysed both with weak pressure signatures. Initially the two frontal cyclones appear to have largely similar characteristics and be of similar horizontal size and depth. For example examination of the potential vorticity at 900mb shows the front defined by an anomalous strip of magnitude 0.4PVU at 00h on the 12th when the first frontal cyclone (L_{R1}) develops, and the same at 00h on the 13th when the

second (L_{R2}) develops. Other diagnostics such as low level vorticity and temperature gradients also appear similar for the two waves. However the two frontal cyclones develop in markedly different ways. Over the 12th L_{R1} increases rapidly in terms of vorticity wavyness $W(\zeta)$ as seen in the vorticity plots (Fig. 3) at 00h and 12h (12th) and graphed in Fig. 5. L_{R2} does not have such an increase in $W(\zeta)$ but becomes less wavy and more 2 dimensional through the 13th.

The hypothesis was that L_{R1} and L_{R2} were being affected in different ways by their environments. To calculate the environmental flow on the front a "frontal box" was defined at each time and the external induced winds calculated (ie. those induced by vorticity and divergence external to the frontal box). The frontal boxes enclosing L_{R1} at 00h and 12h (12th) are outlined in bold in Fig. 3. Figure 4 shows contours of along-front stretching and wind vectors for the 00h frontal box: (a) for the observed winds $\boldsymbol{u}_{obs}$, and (b) for the environmental winds $\boldsymbol{u}_e$. Note there is a great deal of small scale structure in the $\partial v_{obs}/\partial y_f$ field even at this early time when L_{R1} is not that significant; the $\partial v_e/\partial y_f$ field is much smoother and more uniform giving an accurate representation of the background stretching deformation rate (the average over the central half of the box is $0.8\text{x}10^{-5}\text{s}^{-1}$). There are noticeable "end effects" in the environmental stretching due to the cutting of the frontal vorticity strip (by the prescribed frontal box) causing extraneous circulations in the frontal region. By averaging over the centre of the domain their impact can be neglected, see Renfrew (1995) for a longer discussion.

The above analysis was carried out for the 30 hour period of investigation and the results are plotted in Figs. 5 and 6. Figure 5 shows vorticity wavyness $W(\zeta)$ and the along-front averaged vorticity (a measure of frontal strength) and Fig. 6 shows the environmental stretching $\partial v_e/\partial y_f$ for both L_{R1} and L_{R2}. The vorticity wavyness of L_{R1} increases dramatically from 00h to 12h on the 12th, then drops off slowly as the frontal cyclone equilibrates. As shown in nonlinear frontal cyclone simulations (eg. Schär and Davies 1990, Malardel *et al.* 1993) if a frontal cyclone remains barotropic, the instability will quickly saturate and the wave will equilibrate. This appears to have happened for L_{R1} - it is not deep enough to move into a baroclinic growth phase. The rapid increase in $W(\zeta)$ is coincident with a steady

decrease in environmental along-front stretching of the front. The initially large $\partial v_e/\partial y_f$ average of $0.8\times10^{-5}s^{-1}$ has halved to $0.4\times10^{-5}s^{-1}$ by 12h and is negative by 00h on the 13th. In the idealized model of *BT94II*, sustained along-front stretching greater than $0.6\times10^{-5}s^{-1}$ strongly inhibits frontal wave growth. The observed environmental stretching on L_{R1} falls below this value at about 06h. Over the following 6 hours, the vorticity wavyness $W(\xi)$ increases dramatically. Thus, in this case, the observations are qualitatively and quantitatively consistent with their theoretical model, a somewhat surprising result given the idealized nature of the *BT94II* model. In any case, the data for L_{R1} concords with the hypothesis that a weakening environmental stretching rate allows the frontal wave to grow. The data for L_{R2} concords with the converse of this hypothesis - ie. the environmental stretching remains large (around $1\times10^{-5}s^{-1}$ through to 06h on the 13th) and the frontal wave does not develop.

To confirm this scenario along-front profiles of the environmental stretching were plotted; $\partial v_e/\partial y_f$ was calculated by averaging over the central few gridpoints of a longer frontal box enclosing both L_{R1} and L_{R2} at 00h and 06h on the 13th. These show a geographical variation in the environmental stretching along the front - L_{R1} was in a low stretching regime (around $-0.2\times10^{-5}s^{-1}$) and L_{R2} was in a high stretching regime (around $0.8\times10^{-5}s^{-1}$) - the difference being due to their different locations relative to the synoptic systems. The different stretching regimes appear to account for the different evolutions, indeed they accord with the barotropic edge wave modelling results of *BT94II*.

To check whether L_{R1} was growing through barotropic edge wave dynamics derivatives of the perturbation wind field can be examined. The barotropic nature of frontal waves means that diagnostics such as pressure, streamfunction or temperature are often swamped by the basic state (cf. *BT94II*, Fig. 8). Using the attribution method makes it possible - for the first time - to plot the observed wind equivalent of the theoretical perturbation wind derivatives. Figure 7 shows $\partial v_i/\partial y_f$ and $\partial u_i/\partial y_f$ for 12h and 18h on the 12th ($u_i = u_{i\psi} + u_{i\chi}$ the interior induced wind field, analogous to the perturbation wind). Comparing the v-derivatives with Fig. 10 of *BT94II* the patterns are quite similar. The real data $\partial v_i/\partial y_f$ tilts with the shear and about 1½ wavelengths are coherent, there is other

"noise" but the comparison is reasonably convincing. Comparing the u-derivatives (with their Fig. 9) the patterns are less convincing. The cross-front wind is weaker and the derivative pattern is more diffuse. At 12h $\partial u_i/\partial y_f$ is not that similar, but at 18h the $\partial u_i/\partial y_f$ is more like the theory. It is likely that at the earlier time the signature of the weaker cross-front wind component is being masked by other noise; later on the cross-front flow is stronger and the pattern is more obvious. It does seem that barotropic processes are playing an important part in the development of L_{R1}.

Throughout the period the exterior induced horizontal shear $\partial v_e/\partial x_f$ is negative (ie. adverse to the shear of the vorticity front). It falls from around $-0.3\times10^{-5}s^{-1}$ at 00h on the 12th to about zero at 00h on the 13th (for the 900mb level). This decrease in environmental shear implies less suppression of barotropic instabilities (Dritschel 1989, Dritschel *et al.* 1991). However for the shears calculated here (and indeed in all the other tropospheric fronts investigated) this is very much a secondary factor.

In summary L_{R1} moved from an area of strong to weak environmental along-front stretching coincident with a period of rapid growth of the frontal cyclone. This increase in amplitude appears to be due to barotropic edge wave growth. L_{R2} remained in an area of strong environmental stretching and did not amplify. The growth mechanism was suppressed by the configuration of the synoptic systems. So the geographical distribution of stretching deformation was crucial in determining the different frontal cyclone evolutions.

(b) Case of 1st-2nd February 1994

At 18h on the 1st February 1994 (Fig. 8) a number of frontal cyclones are shown on a wavy cold front extending across Northern Europe and the North Atlantic. Two of these, L_Q and L_R, are shown with closed isobars and appear approximately the same size and magnitude. Examination of PV at 900mb indicates them to be similar in structure and size. However their subsequent evolutions are totally different: L_Q remains an ordinary frontal cyclone - surface pressure staying around 1000mb before eventually saturating; whereas

L_R deepens explosively (47mb in 42 hours) into a full blown storm. This is a particularly interesting case as the UK Met. Office 36 hour forecast was very poor - missing the explosive growth of L_R and instead deepening both L_Q and L_R to a moderate extent and with different locations (see Renfrew 1995).

The rapid deepening and increase in scale of L_R signifies a transition from a barotropic growth process to a baroclinic one (see Malardel *et al.* 1993), something which did not happen for L_Q. It was hypothesised that the two frontal cyclones differing evolutions may have been due to different environmental flows. So the attribution technique was used to investigate the period from 00h on the 1st to 12h on the 2nd. On investigation it was found that both frontal cyclones were undergoing low stretching deformation through the 1st and 2nd (L_Q negative, around $-0.2\times10^{-5}s^{-1}$, and L_R positive, at $0\text{-}0.3\times10^{-5}s^{-1}$). These values are well below the critical mark of *BT94II* and so barotropic instabilities should be free to grow, although note the effect of negative stretching on barotropic instabilities is an area of continuing research (Dritschel *et al.* 1991, D.J. Parker personal communication). An examination of the Bishop (1993II) "f_1" case (for a mid-latitude baroclinic zone, cf. his Figs. 2 & 3) indicates this magnitude of strain also has a negligible effect on baroclinic growth. So the calculated environmental stretching should allow either (or both) L_Q and L_R to grow barotropically or baroclinically; the action of the environmental flow does not seem to make the crucial difference in their developments.

Instead a closer examination of the size and depth of the two frontal cyclones reveals that L_R was marginally larger than L_Q - indeed large enough to `feel' the upper boundary, ie. larger than the Rossby radius (cf. Malardel *et al.* 1993). Along-front cross-sections of potential vorticity (Fig. 9) reveal that while L_Q was just too shallow to phase lock with a tropopause undulation, L_R was deep enough to phase lock with a significantly more pronounced tropopause fold. Isentropic advection of PV from the stratosphere into the lower troposphere allowed a rapid deepening of L_R through baroclinic processes. This fits the scenario put forward by Thorncroft and Hoskins (1990) to explain the explosive growth of frontal cyclones: upper level reinforcement of an existing low level anomaly.

The lack of large scale horizontal deformation motivates the comparison of this case

with the horizontal shearing frontogenesis model of Joly and Thorpe (1991). Calculations of the along-front temperature gradient due to the large scale flow were made, via the 2 level balance approach - see Appendix. It was found that the temperature gradients were not that strong on average. For example using the 900mb and 700mb levels the along-front temperature gradient due to the environmental flow was about 0.1K/100km at L_R, and -0.2K/100km at L_Q at 18h on the 1st. This would imply a small value of the s^2 parameter ($s^2 = g/\theta_0 . \partial\theta/\partial y_f$) for this time. In the Eady problem and in horizontal shearing frontogenesis models a typical value of s^2 is taken to correspond to an along-front gradient of ±0.5-1K/100km. The slow frontogenesis implied here would favour the amplification of 1000km-scale waves in the shearing frontogenesis model of Joly and Thorpe (1991). From these 1000km scale waves it seems one of them, L_R, becomes large enough to couple with a pronounced tropopause undulation and deepen explosively, whereas the other saturates through nonlinear processes.

(c) Case of 26th November 1994

This case appeared to be a classic Bergeron deformation forced front - the surface pressure analysis (Fig. 10) indicates a low level confluence flow on the front trailing southwards from Low_L (954mb). The front was quite shallow but had a well defined signature in low level vorticity (Fig. 11), potential temperature and also in PV (magnitude 1PVU at 900mb). So instabilities of the type modelled in Joly and Thorpe (1990) or Schär and Davies (1990) may be possible. However the along-front stretching due to the environmental flow was calculated to be extremely strong: deformation rates between 1.2 & $1.6 \times 10^{-5} s^{-1}$ for the 900mb and 700mb u_e winds (00h to 12h). These are larger than the $1 \times 10^{-5} s^{-1}$ value usually prescribed in frontogenesis theory, and significantly greater than the critical values of *BT94II*.

From the *BT94II* model it is actually possible to calculate a minimum strain rate (α_{min}) necessary to suppress *all* barotropic frontal wave growth. For a valid comparison to be made it is necessary to check that the observed case and the idealisation are alike. So

diagnostics of moist PV and relative humidity were examined to confirm that the front was close to moist neutrality ($PV_e \approx 0$) and saturated (the relative humidity was over 95%). Hence the following equation could be employed:

$$\alpha_{min} = \frac{f}{4}\left(\frac{\zeta_4 - \zeta_2}{\zeta_4 + \zeta_2}\right) e^{-2\mu} \qquad (8)$$

where ζ_4 is the absolute vorticity of the strip, ζ_2 is the absolute vorticity of the neighbouring anticyclonic region and 2μ is the nondimensional wavenumber of the disturbance. Taking $\zeta_4 = f + 1.0\text{x}10^{-4}\text{s}^{-1}$ and $\zeta_2 = f - 0.3\text{x}10^{-4}\text{s}^{-1}$ (Fig. 11) with $f = 1.0\text{x}10^{-4}\text{s}^{-1}$; then setting $2\mu=0$ (ie. an infinite wavelength) sets an upper limit for α_{min}, so at 00h $\alpha_{min} = 1.2\text{x}10^{-5}\text{s}^{-1}$ for complete wave suppression. The measured value of $\partial v_e/\partial y_f$ is $1.6\text{x}10^{-5}\text{s}^{-1}$ so on this front the theory predicts that all waves should be suppressed. Note in reality 2μ is non-zero and instead the wavelength of any instability is limited by the width and length of the vorticity strip. An estimate of these at 00h gives a width of 250km and a length of 2500km, so $e^{-2\mu} = 0.63$ and $\alpha_{min} = 0.75\text{x}10^{-5}\text{s}^{-1}$ less than half the measured rate. However one must also bear consider that the magnitude of the vorticity of the frontal strip will not be resolved particularly well in the forecast model. If the relative vorticity of the strip is doubled ($\zeta_4 = f + 2.0\text{x}10^{-4}\text{s}^{-1}$), then α_{min} would be $0.98\text{x}10^{-5}\text{s}^{-1}$ for the estimated strip dimensions - also much less than the measured rate. Over the next 12 hours the front remains fairly 2 dimensional, the vorticity wavyness $W(\xi)$ decreases from $0.7\text{x}10^{-4}\text{s}^{-1}$ at 00h to $0.5\text{x}10^{-4}\text{s}^{-1}$ then $0.3\text{x}10^{-4}\text{s}^{-1}$ at 12h, as predicted the wave growth is suppressed by the strong deformation flow.

(d) Case of 20th-22nd January 1995

A common scenario observed in the North Atlantic sector is when a mesoscale secondary cyclone `runs around' the southern flank of a parent synoptic scale cyclone (the "Icelandic Low") and then deepens as it moves into the parent system, eventually

becoming a parent cyclone itself (eg. Browning and Roberts 1994, Browning *et al.* 1995). This scenario occurred four times in secession in January 1995 and two of these events are investigated here.

Figure 12 shows surface pressure analyses at 18h on the 21st: L_A has moved around L_S and is now a parent cyclone with a col wave L_U growing on its front. L_A appeared to have been driven by an upper level PV tongue and only deepened when the tongue moved over a low level baroclinic zone about 00h on the 21st. L_U appeared primarily a low level feature with a strong low level PV strip (1.2PVU at 00h, 21st) but with the added complication of a meandering upper level tropopause anomaly (marked as a trough on Fig. 12) which also seemed to play a role in the low level growth.

Figure 13 shows the vorticity wavyness $W(\xi)$ and the along-front average vorticity over the period, for both L_A and L_U. There is an increase in $W(\xi)$ and along-front averaged vorticity for L_A around 00h on the 21st. This is coincident with the movement of the upper PV tongue over the low level baroclinic zone. The growth process is envisaged as primarily a baroclinic interaction between the upper tongue and the low level baroclinicity similar to that modelled in Thorncroft and Hoskins (1990). Initially it would appear there is not a strong enough low level PV strip at L_A for barotropic instabilities to play an important role. However the increase in amplitude of L_A does coincide with a decrease in environmental along-front stretching (Fig. 14) which would encourage barotropic instabilities - adding to the primary baroclinic instability.

Turning to L_U Fig. 13 illustrates the relative strength of the vorticity strip at L_U throughout the examination period. The frontal cyclone undergoes a rapid amplification from 06h to 12h (on the 21st) with a more gradual increase over a further 24 hours. The increase in $W(\xi)$ is coincident with a decrease in the environmental along-front stretching of L_U (Fig. 14). The amplification of L_U occurs when $\partial v_e/\partial y_f$ falls from $1.2\text{x}10^{-5}\text{s}^{-1}$ to around $0.8\text{x}10^{-5}\text{s}^{-1}$, this is towards the high end of the critical threshold zone for barotropic instabilities (cf. *BT94II*). However the temporal change in stretching of L_U is striking, with a clear move from high to low stretching over 24 hours. This suggests that as well as the importance of the actual strain rate, the time rate of change of strain is also important. An

extension of the *BT94II* model could be used to investigate this problem.

To confirm whether L_U was growing through barotropic processes the perturbation wind derivatives $\partial v_f/\partial y_f$ and $\partial u_f/\partial y_f$ were examined [as described for case (a) - Fig. 7]. These showed the tilts with and against the vorticity strip shear indicative of barotropic growth, however the patterns were not as distinct as those in case (a). To ascertain whether baroclinic growth processes were present cross-sections of PV along the front were examined, and an along-front ('westerly') tilt with height was visible between the low level L_U and the meandering upper level tropopause anomaly. However the magnitude of the tilting PV anomaly was not that great (0.8PVU), so a vertical phase locking between the low level PV and the tropopause anomaly does not appear the sole amplifying process. The low level PV strip has a maximum around 1-2km in height, and so internal baroclinic growth may also be occurring (cf Charney and Stern 1962). It seems that a complex mixture of all the above growth mechanisms is responsible for the amplification of L_U. The drop in environmental stretching appears to have played a key part in allowing barotropic edge wave growth.

4. DISCUSSION

To try and synthesize all the case study analyses into an overall picture, a regime diagram has been developed. Figure 15 plots the rate of change of vorticity wavyness $(dW(\xi)/dt)$ against environmental along-front stretching $(\partial v_e/\partial y_f)$ for all the cases investigated. The rate of change of $W(\xi)$ is a measure of the *growth* of a frontal cyclone. The along-front stretching rates are normalised by α_{min}, the minimum strain rate required to suppress all barotropic frontal wave growth (cf Eq. 8 and *BT94II*). This normalisation accounts for the differing strengths of the vorticity strips in each case; clearly if the vorticity strip were not that anomalous then significant wave amplification through edge wave dynamics could not be expected.

The regime diagram allows the testing of a hypothesis suggested by the theoretical modelling work of *BT94II*, Dritschel *et al.* (1991) and Renfrew (1995). The hypothesis is that

sustained, strong along-front stretching is a sufficient condition for barotropic frontal stability; weak along-front stretching is a necessary condition for barotropic frontal instability. To test the hypothesis all the points from all the cases are plotted, except those points where the frontal cyclone has started to saturate through nonlinear processes (ie. where the cyclone has left the developing stage). This is judged to have happened if *(i)* the frontal cyclone has reached a finite amplitude size (defined by $W(\xi)$ greater than $1.0\times10^{-4}s^{-1}$); and *(ii)* the frontal cyclone is at the end of its lifecycle ($W(\xi)$ is monotonically decreasing). This eliminates 9 data points, leaving 20 points. The error bars indicate the maximum variation found in the sensitivity study of Renfrew (1995), where the frontal box size and angle were varied by small amounts (see section 2). Note the rate of change of $W(\xi)$ is calculated by differencing the 6 hourly sampled values (ie. by differentiating the curves plotted in Figs. 5 or 13 for example). The corresponding $\partial v_e/\partial y_f$ is taken as the mean of the two stretching values of those times, normalised by α_{min}. The α_{min} is calculated for each time, with $2\mu=0$ (ie. infinite wavelength); recall this provides an upper bound for α_{min}. This means the *critical threshold* for barotropic wave suppression is a normalised stretching rate of 1; *weak* stretching is less than 1, and *strong* stretching is greater than 1. To satisfy the hypothesis above, all the points in Fig. 15 should lie in either the SE quadrant (strong stretching & no growth) or the NW quadrant (weak stretching & growth), with the dividing line at 1. All but 4 points lie in these two quadrants. The majority of the plotted times the cyclones are *growing* in an environment of low stretching deformation. At 5 times [L_{R2} case (a), case (c) and L_A case (d)] the cyclones are *decaying* in an environment of strong stretching deformation. There are 2 points in the SW quadrant where the frontal cyclone has undergone a small decrease in amplitude as measured by vorticity wavyness, however as illustrated in Fig. 13 [at 00h for L_U case (d)] this is only a small decrease in a clear overall trend, and within the error bars is near zero. There are also 2 points in the NE quadrant; the 2 "o" points of L_U case (d). These are from 03h and 09h on the 21st January, early in the growth phase of L_U. It was noted that the stretching was falling at these times, but was still large, suggesting the idea that the time rate of change of stretching may be important - as well as the actual stretching rate. In addition, there appeared to be a baroclinic element in the growth of L_U; it may be

that this baroclinic element is providing a contribution at these times which overwhelms the suppressing effects of the strain.

In conclusion it appears the hypothesis relating barotropic frontal cyclone growth and environmental stretching is largely born out. The majority of data points fit the hypothesis with 4 exceptions. In dimensional units the theoretical *critical threshold* is in the range 0.5 to $1.2 \times 10^{-5} s^{-1}$ for the analysed cases (taking $2\mu=0$, ie. an upper bound). For a typical strength vorticity front (an along-front average vorticity of about $1 \times 10^{-4} s^{-1}$), a critical threshold around $0.8 \times 10^{-5} s^{-1}$ seems to fit the observed results.

5. CONCLUSIONS

It has become clear that the role of stretching deformation due to the synoptic environment of a front is crucial in the evolution of any instabilities on the front. A change of stretching deformation, over time or location, can differentiate growth from suppression of frontal waves. For barotropic instabilities a hypothesis suggested by theoretical modelling work is largely born out - as summarised in Fig. 15. If the stretching deformation of the front is strong (relative to the vorticity strip), then barotropic instabilities are unlikely to grow. If the stretching deformation of the front is weak, then barotropic instabilities will be free to grow - if they have the means to do so (in the form of edge waves on a low level potential vorticity strip). In certain cases the observed `perturbation' wind derivatives were examined and compared to model perturbation wind derivatives, adding evidence that the frontal cyclone's growth was through barotropic edge wave amplification.

In addition this study has provided a range of values for the commonly prescribed deformation frontogenesis parameter α, by calculating the environmental along-front stretching $\partial v_e / \partial y_f$. The calculated α ranged from $-0.5 \times 10^{-5} s^{-1}$ to $1.6 \times 10^{-5} s^{-1}$; the same order of magnitude as the standard theoretical value, but with a range that is significant for potential barotropic instability growth and indeed frontogenesis models. A two level extension of the attribution technique (see the Appendix) allowed the calculation of a

range of values for the shearing frontogenesis parameter s^2 corresponding to -0.4K/100km to 0.1K/100km, slightly less than the values generally taken in frontogenesis studies.

The attribution technique developed here is based on the vorticity and divergence attribution of Bishop (1996I&II) and so is suited to phenomenon where 2 dimensional vorticity dynamics are important. For example when barotropic processes appear to play an important role, such as in frontal cyclones or tropical cyclones. An interesting extension of the technique would be to examine vorticity on a (quasi-horizontal) isentropic surface. This would be appropriate for looking at upper tropospheric levels where flow is generally on isentropic surfaces. In this case isentropic vorticity is the key quantity, and this is equal to potential vorticity, scaled by a density factor. This variant would be useful in examining the stability of upper tropospheric PV tongues and filaments, such as those seen in high resolution modelling studies (eg. Methven 1996) and on a broader scale in observed forecast model data (eg. Hoskins *et al.* 1985). An interesting discussion point is the application of this work to forecasting, and this is addressed in Renfrew (1996).

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APPENDIX

Vorticity and divergence attribution can only be used on one level of data at one time, however without knowing the vertical structure of a front it is not possible to establish the amount of shear frontogenesis the front is undergoing. The classic Hoskins and Bretherton (1972) shear front has a prescribed along-front temperature gradient

(equivalently a geostrophic vertical shear) given by the parameter $s^2 = -g/\theta_0 . \partial\theta/\partial y_f$. This cannot be estimated by a single level analysis, as if the temperature gradient is directly measured from observed data, local variations will mask the large scale value. Hence to measure s^2 accurately in real data the large scale induced temperature gradient must be found. To do this a two-level attribution method is employed: the attribution method is carried out for the 900mb and 700mb levels for example. Assuming a balance condition it is possible to retrieve the geopotential (ϕ) corresponding to a certain wind field. For example using geostrophic balance:

$$\phi = \frac{\psi}{f} , \tag{9}$$

where ψ is a streamfunction and f the Coriolis parameter (taken as constant). Differencing ϕ in the vertical via

$$\frac{\partial\phi}{\partial z} = \frac{g\theta}{\theta_0} = b \tag{10}$$

(assuming hydrostatic balance) will yield the potential temperature field corresponding to that streamfunction. Unfortunately when the streamfunction is found at more than one level an arbitrary constant renders the actual value of the geopotential or potential temperature meaningless. However the main interests are the thermal gradients not the actual temperature itself, so differentiating in the horizontal:

$$\frac{\partial}{\partial y_f}\left(\frac{\partial\phi}{\partial z}\right) = -\frac{g}{\theta_0}\frac{\partial\theta}{\partial y_f} \quad \textit{and} \quad \frac{\partial}{\partial x_f}\left(\frac{\partial\phi}{\partial z}\right) = \frac{g}{\theta_0}\frac{\partial\theta}{\partial x_f} \tag{11}$$

the unknown constants are dispensed with, and the temperature gradients corresponding to certain parts of the flow are retrieved. The streamfunction used in the balance approximation (9) may be the total streamfunction, or the streamfunction due to frontal vorticity ψ_i, or the streamfunction due to non-frontal vorticity ψ_e. Using ψ_e one would expect to retrieve the large scale along and across-front temperature gradients. So the

prescribed values of s^2 in Hoskins and Bretherton (1972) or Keyser and Pecnick (1987) would be found.

There are several approximations used in the above method:

• using only the streamfunction in the balance approximation means only considering the vorticity induced winds. The much smaller divergence induced winds are neglected. This is reasonable for a dominantly rotational flow.

• the simple geostrophic balance approximation $\phi=\psi/f$ is used as one would expect the large scale flow to be in approximate geostrophic balance, so to find the large scale temperature gradients this balance is adequate. This is less justified in finding the frontal temperature gradients.

• hydrostatic balance is employed. This is a negligible approximation given the above.

Note the two-level method requires the fronts have a reasonably coherent vertical structure in vorticity. If cross-front thermal wind balance is maintained then this will be the case, and in most observed fronts this appears accurate (see Thorpe and Clough 1991 for example). The two level approximation can be used to calculate measures of deformation and shearing frontogenesis based on the environmental winds and frontal temperature gradients; and frontal winds and environmental temperature gradients respectively - see Renfrew (1995) for details.

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FIGURE CAPTIONS

Figure 1 Surface pressure analysis at 12GMT (or 12h) on the 12th November 1994 for the North Atlantic - Western Europe sector. Isobars are every 4mb and lines of latitude and longtitude every 10°. Two frontal cyclones are analysed at about 30°W, 45°N and 40°W, 40°N on the cold front. From Weather.

Figure 2 A schematic of the frontal box.

Figure 3 Relative vorticity plots at **(a)** 00h hand **(b)** 12h on the 12th November 1994 at the 900mb level. The frontal boxes are outlined in bold. The contour interval is $0.4\text{x}10^{-4}\text{s}^{-1}$, with lines of latitude and longtitude drawn every 10° and the axes labelled in km.

Figure 4 Contours of along-front stretching for **(a)** the observed winds and **(b)** the environmental winds, at 00h on the 12th November 1994, for the frontal box indicated in Fig. 3. Also shown are vectors of the corresponding winds ie. u_{obs} and u_e respectively. The contour interval is $0.5\text{x}10^{-5}\text{s}^{-1}$ and the scaling arrows for the winds are 28ms^{-1}. The axes labelling is in km.

Figure 5 Vorticity wavyness $W(\xi)$ and the maximum along-front averaged vorticity for the

30 hour period of investigation for L_{R1} and L_{R2}, case (a). The along-front average is over the full frontal box.

Figure 6 Along-front environmental stretching rates at the 900mb and 700mb levels for L_{R1} and L_{R2}, case (a). The plotted values are averages over the central half of the frontal box.

Figure 7 Contours of the along-front frontal-induced wind derivatives $\partial v_i/\partial y_f$ (left) at **(a)** 12h & **(b)** 18h, and $\partial u_i/\partial y_f$ (right) at **(c)** 12h & **(d)** 18h on 12th Nov. 1994. The vectors are the corresponding u_i winds. The contour interval is $1\times10^{-5}s^{-1}$ the same as Fig. 10 in *BT94II*. The scaling arrow is $15ms^{-1}$. The frontal box for 12h is as shown in Fig. 3.

Figure 8 Surface pressure analysis at 18h on the 1st February 1994 for the North Atlantic sector. Isobars are every 4mb. L_Q and L_R are analysed with closed isobars at this time. From the UK Met. Office.

Figure 9 A cross-section of potential vorticity (colours) and isentropes (contours) along the front through L_R (left) and L_Q (right) at 12h on the 2nd February. The colours indicate PV values (in PVU) and the isentropes are every 5K as labelled.

Figure 10 Surface pressure analysis for 00h on the 26th November 1994 from the UK Met. Office. Isobars are every 4mb.

Figure 11 Relative vorticity at 900mb for 00h on the 26th November 1994. As in Fig. 3 the frontal box is outlined in bold and the contour interval is $0.4\times10^{-4}s^{-1}$.

Figure 12 Surface pressure analysis for 18h on the 21st January 1995 from the UK Met. Office. Isobars are every 4mb.

Figure 13 Vorticity wavyness $W(\xi)$ and the maximum along-front averaged vorticity for the 60 hour period of investigation for L_A and L_U , case (d). The along-front average is taken over individual frontal boxes for each cyclone.

Figure 14 Along-front environmental stretching rates at 900mb for L_A and L_U , case (d). The plotted values are averages over the central half of the frontal box.

Figure 15 A regime diagram showing growth against along-front stretching. The growth is measured as the rate of change of vorticity wavyness $W(\xi)$ [units $x10^{-9}s^{-2}$] and the stretching is measured as $\partial v_e/\partial y_f$ normalised by α_{min} (the critical threshold for suppression of all barotropic waves). Data points are plotted for all four cases. The error bars indicate the maximum errors found in the sensitivity study of case (a), as discussed in Renfrew (1995) and mentioned in section 2. In the NW quadrant waves are growing in weak environmental stretching deformation; in the SE quadrant wave growth is suppressed by the strong environmental stretching deformation.

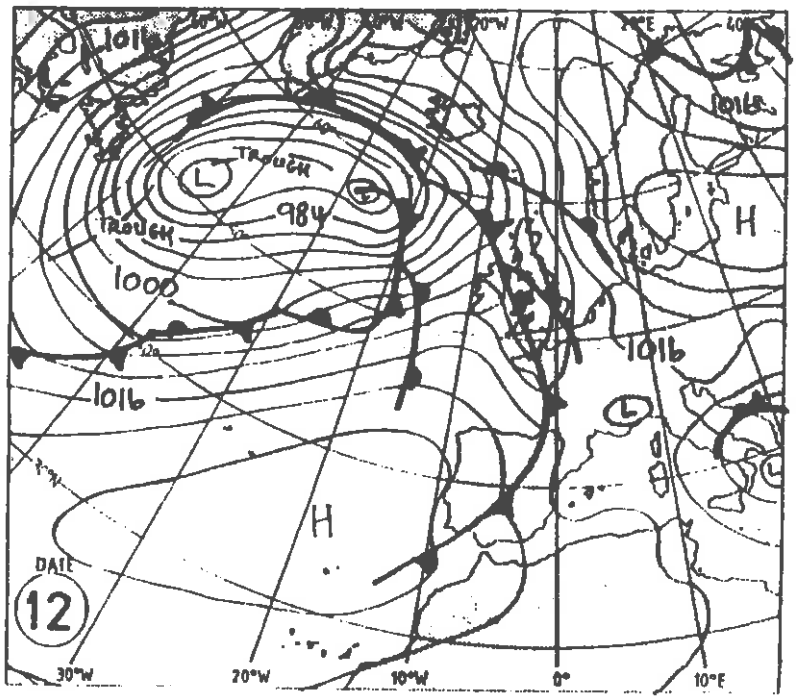
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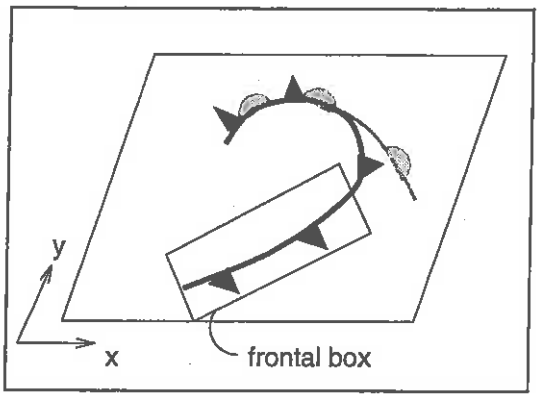
Figure 1

Figure 2



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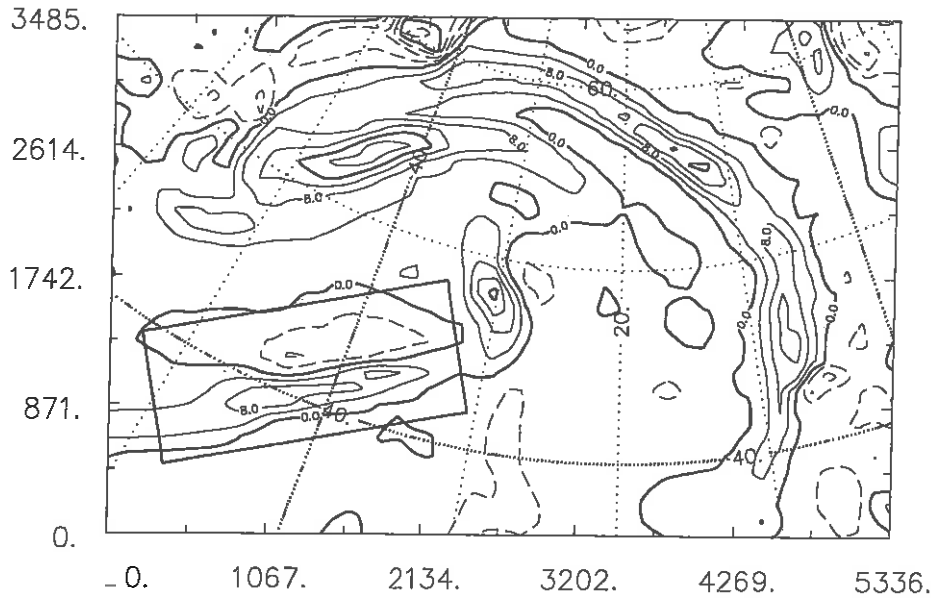
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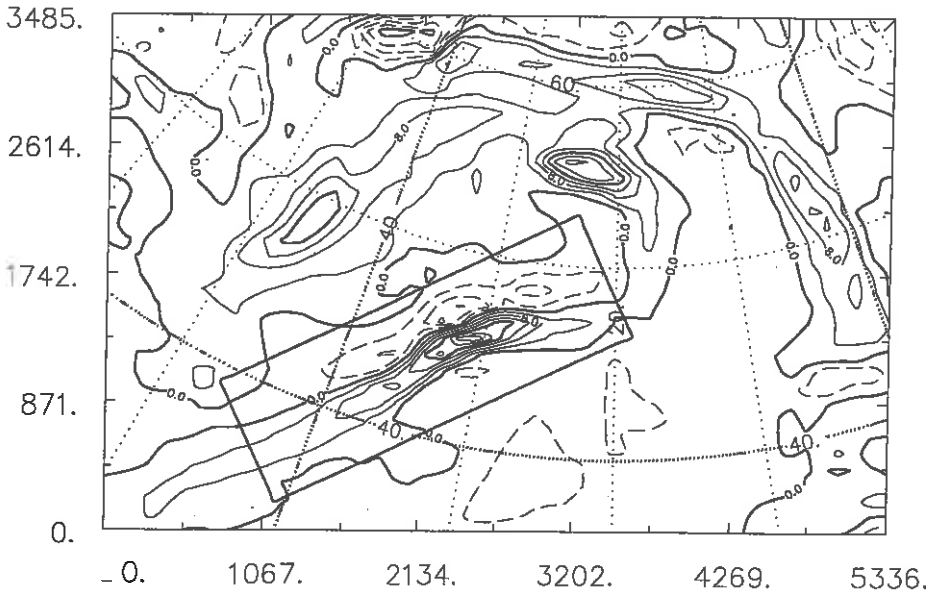
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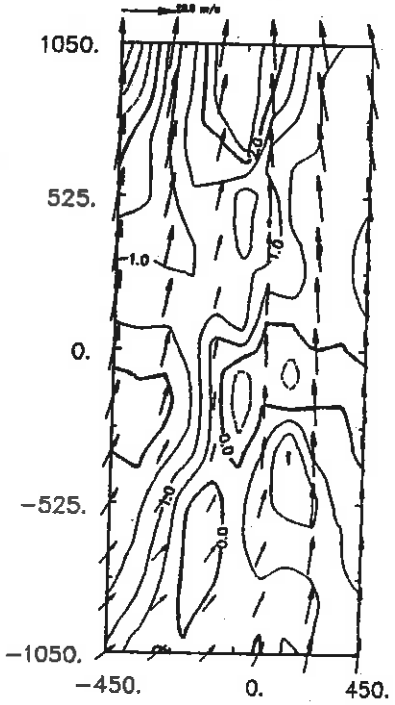
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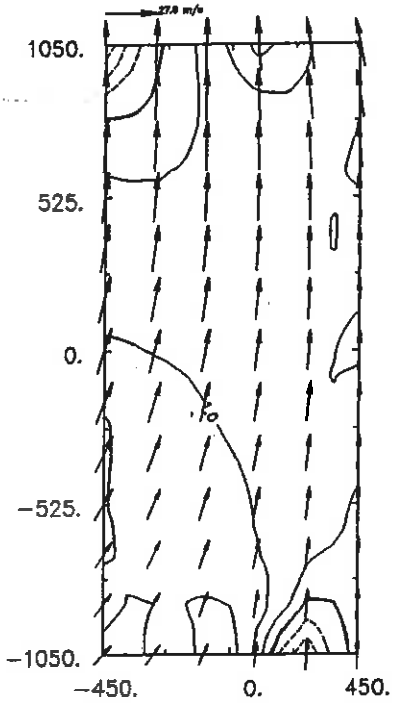
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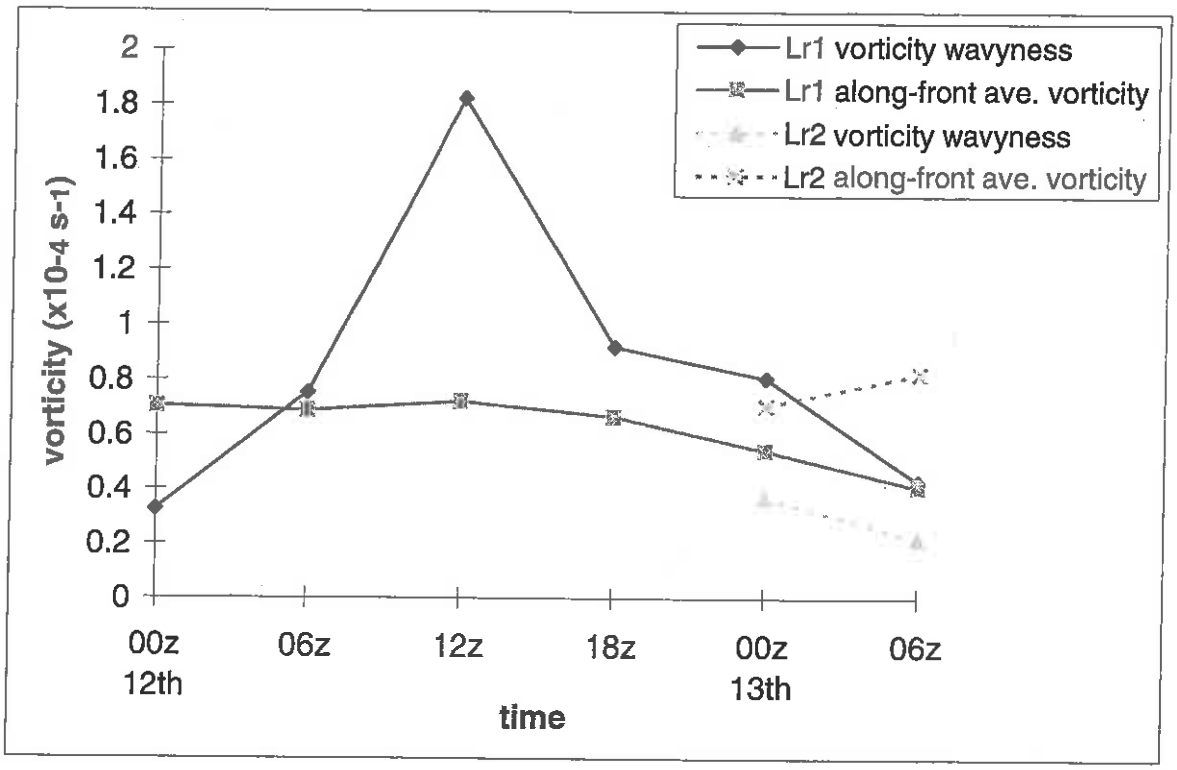

Lr1 vorticity wavyness
Lr1 along-front ave. vorticity
Lr2 vorticity wavyness
Lr2 along-front ave. vorticity
vorticity (x10-4 s-1)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
00z 12th
06z
12z
18z
00z 13th
06z
time

Fig5

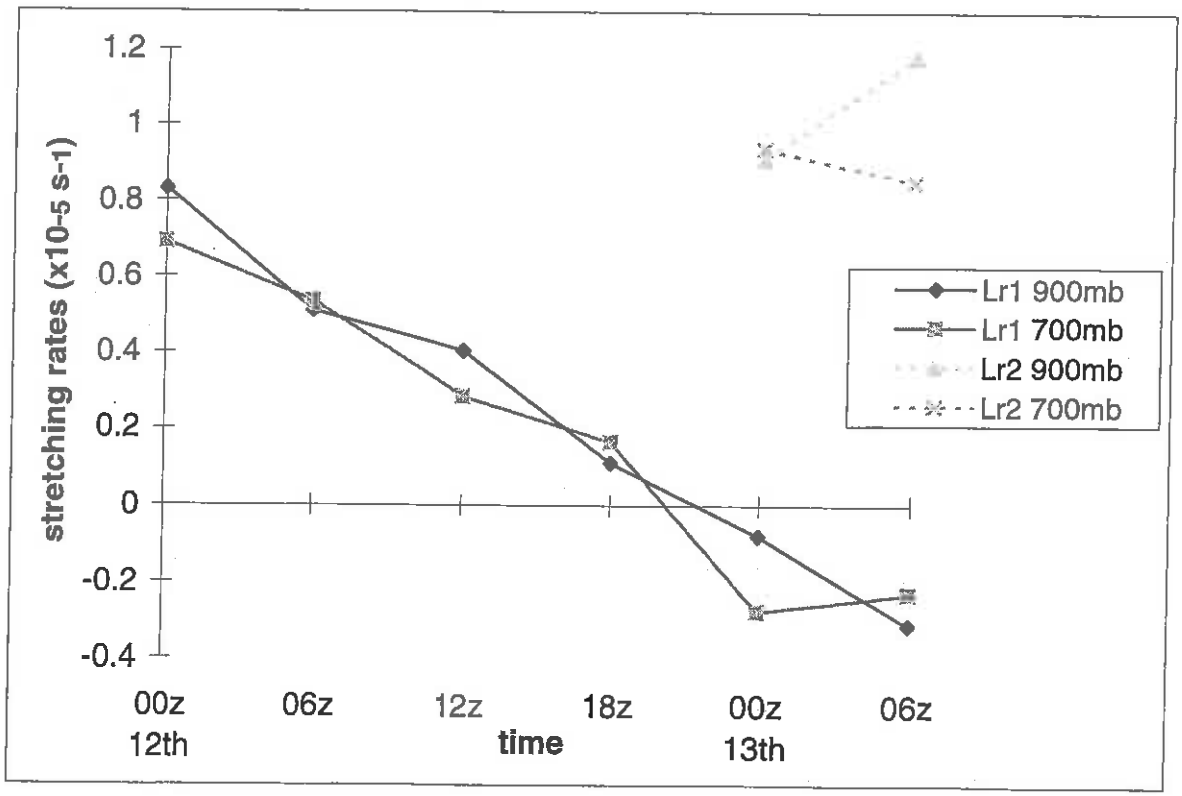

Lr1 900mb
Lr1 700mb
Lr2 900mb
Lr2 700mb
stretching rates (x10-5 s-1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
00z 12th
06z
12z
18z
00z 13th
06z
time

Fig6

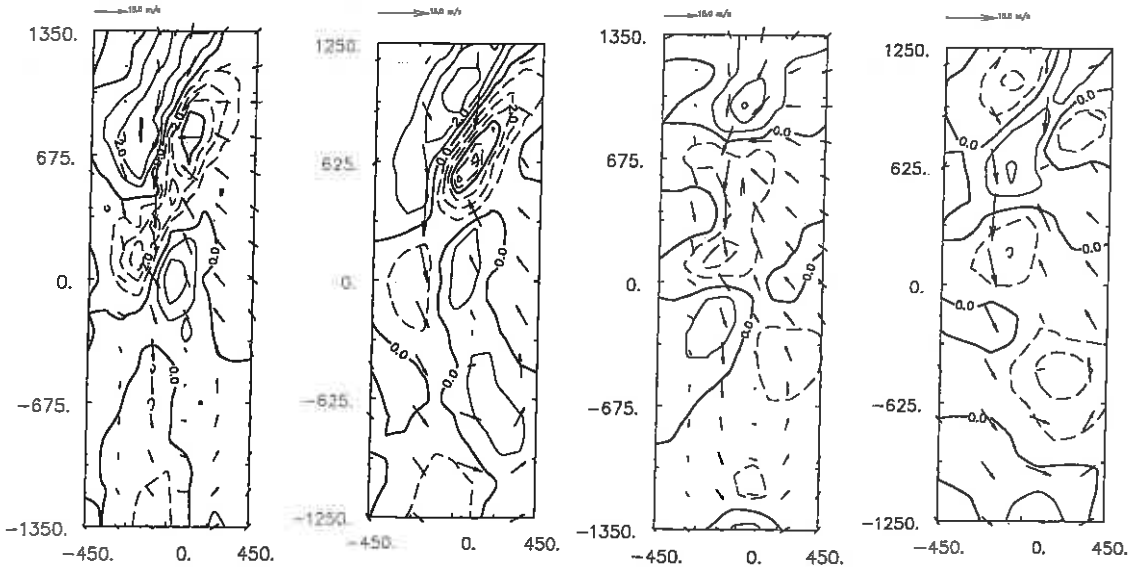

1350.
675.
0.
−675.
−1350.
−450.
0.
450.
1250.
625.
0.
−625.
−1250.
−450.
0.
450.
1350.
675.
0.
−675.
−1350.
−450.
0.
450.
1250.
625.
0.
−625.
−1250.
−450.
0.
450.

Fig 7 (a) (b) (c) (d)

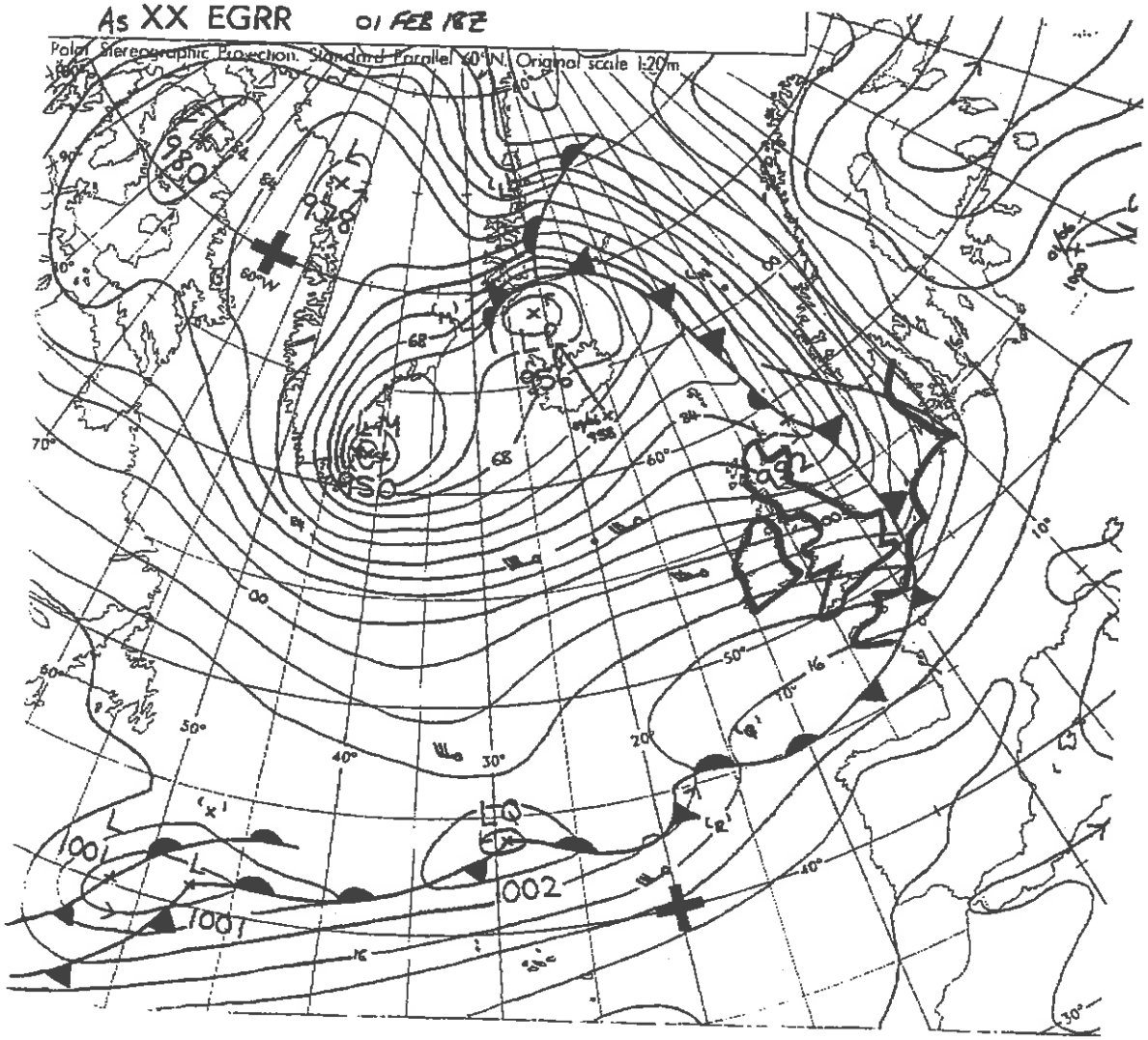
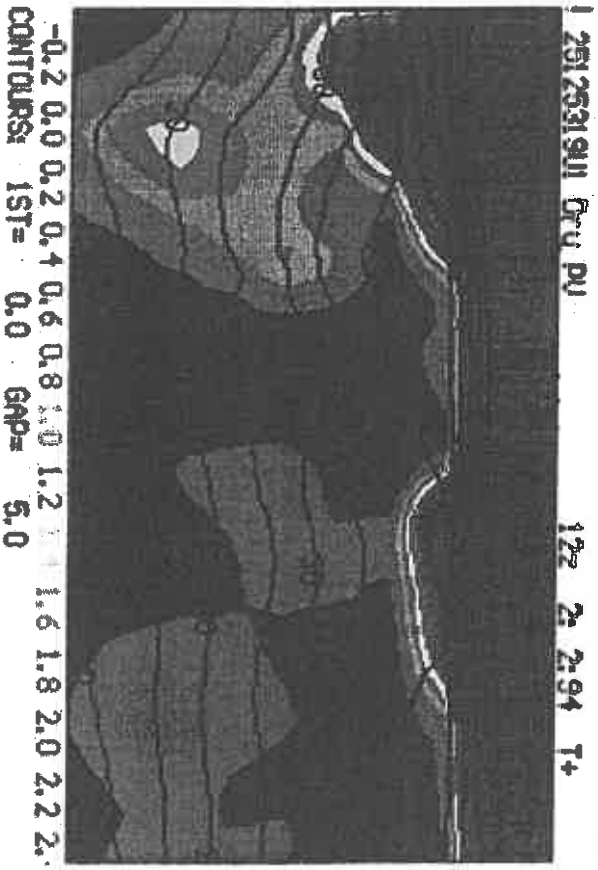

As XX EGRR 01 FEB 82
Polar Stereographic Projection. Standard Parallel 60° N. Original scale 1:20m.
1001
1001
002

Fig8

Fig 9



CONTOURS: 1ST= 0.0 GAP= 5.0
-0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.6 1.8 2.0 2.2
12z 2: 2.94 T+

Fig. 9

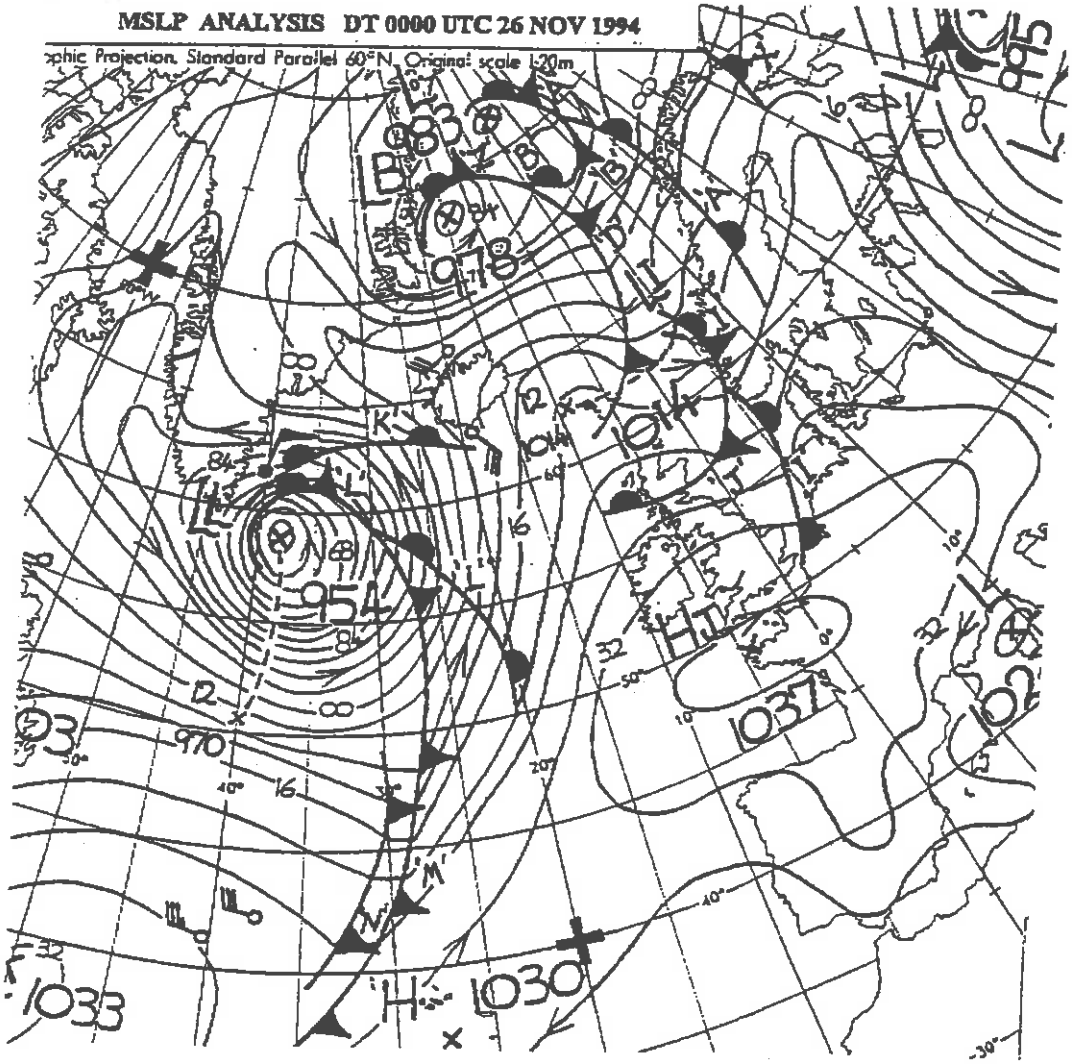

MSLP ANALYSIS DT 0000 UTC 26 NOV 1994
phic Projection. Standard Parallel 60°N. Original scale 1:20m
978
954
1014
1037
1030
1033
970

Fig 10

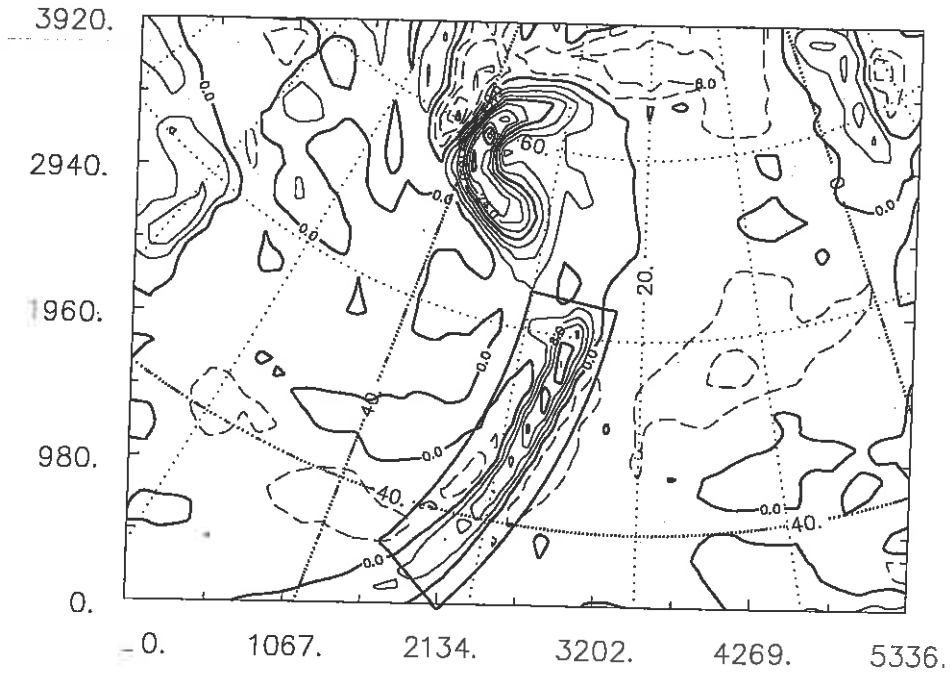


Fig 11

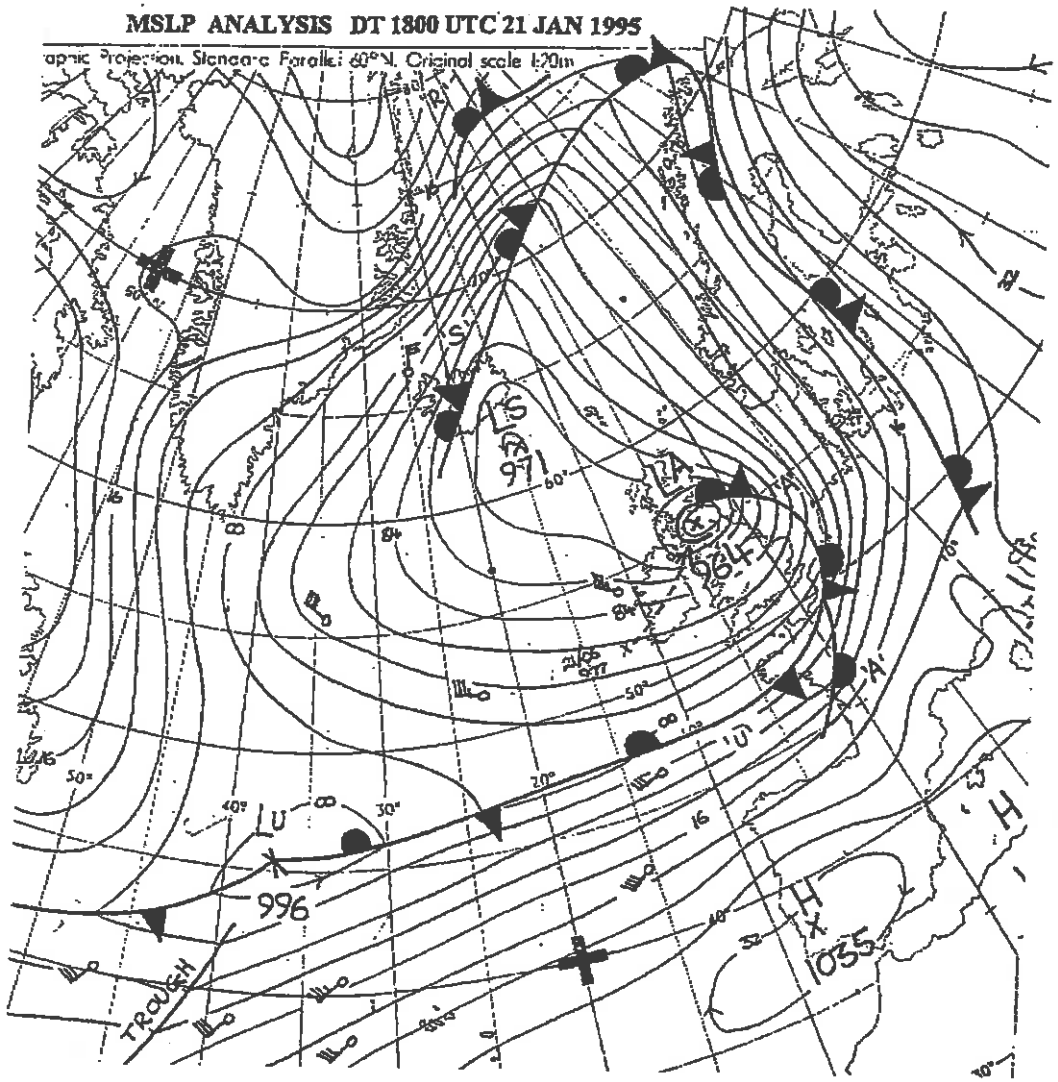

MSLP ANALYSIS DT 1800 UTC 21 JAN 1995
LS
971
LA
964
LU
996
TROUGH
H
1035

Fig 12

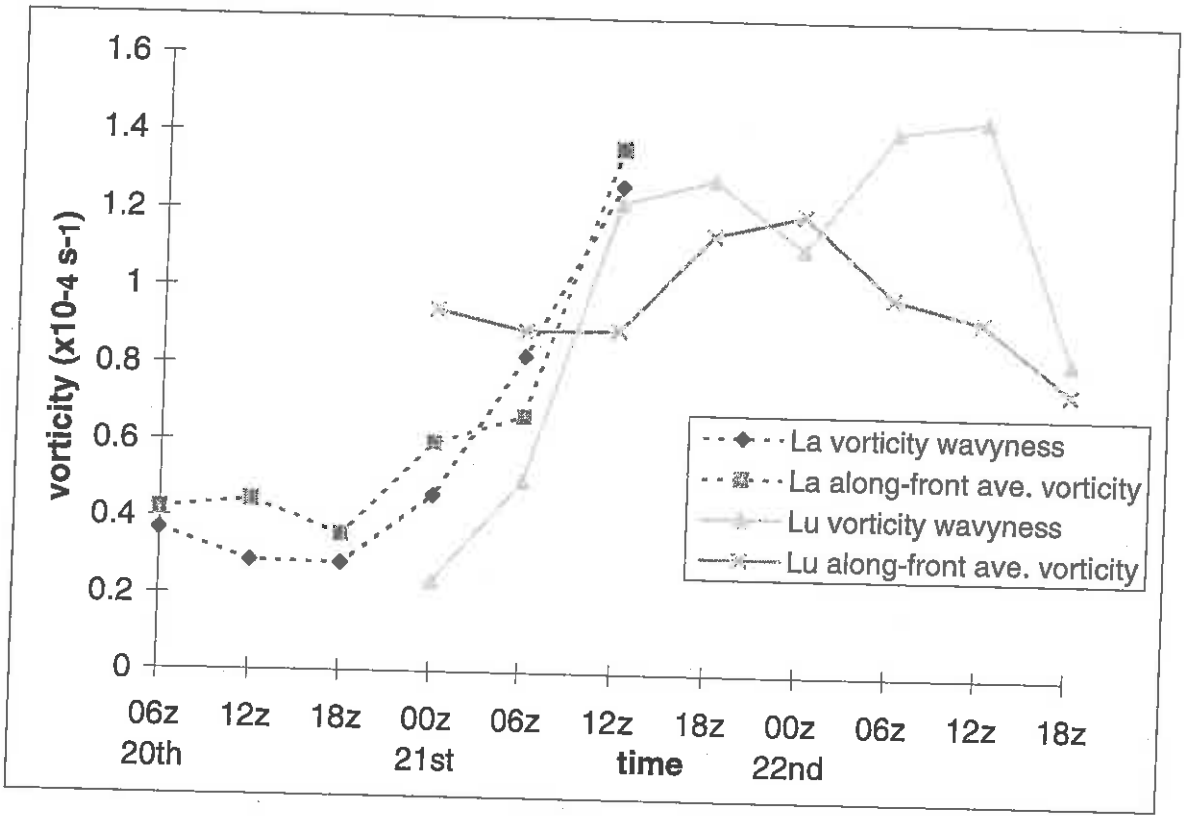

1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
vorticity (x10-4 s-1)
La vorticity wavyness
La along-front ave. vorticity
Lu vorticity wavyness
Lu along-front ave. vorticity
06z 20th
12z
18z
00z 21st
06z
12z
18z
00z 22nd
06z
12z
18z
time

Fig13

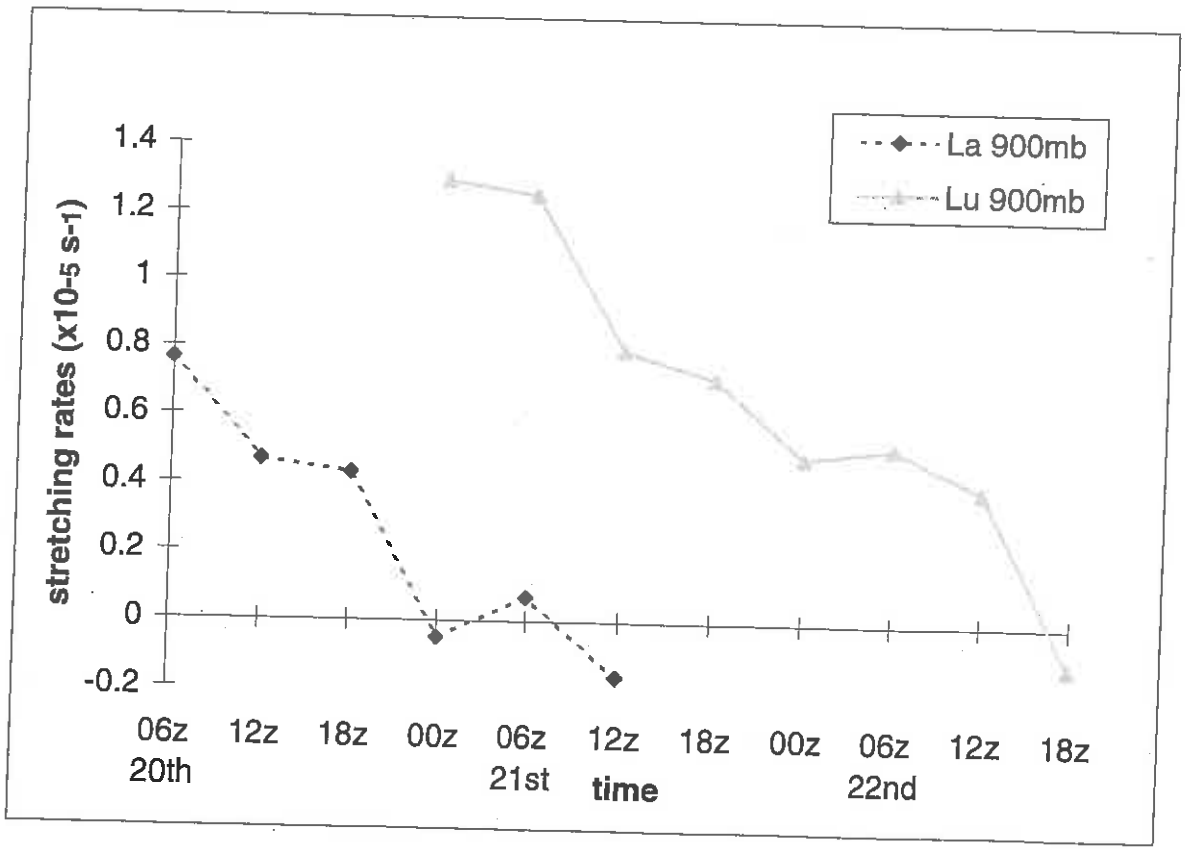

1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
stretching rates (x10-5 s-1)
La 900mb
Lu 900mb
06z 20th
12z
18z
00z
06z 21st
12z
18z
00z
06z 22nd
12z
18z
time

Fig14

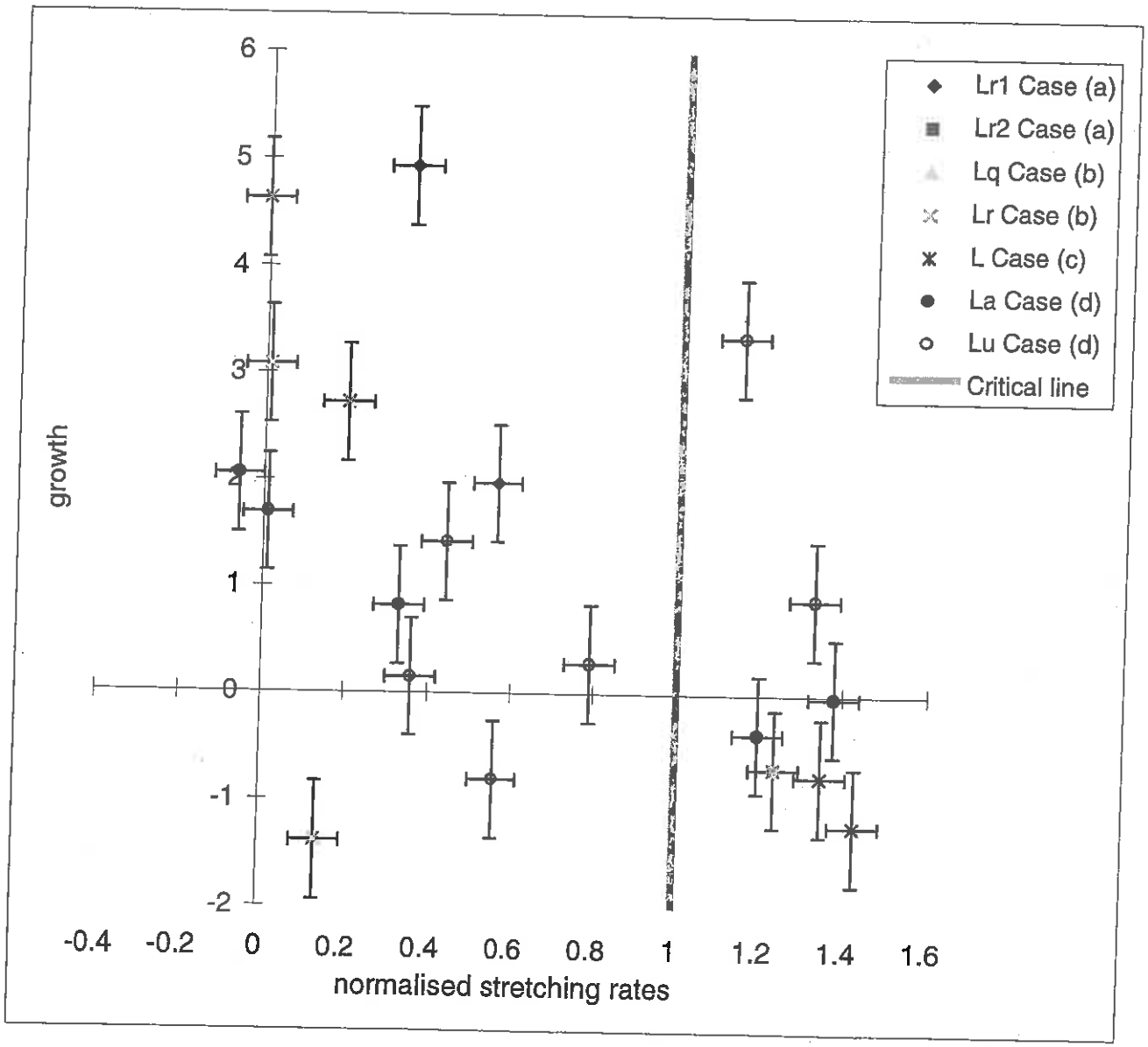

Lr1 Case (a)
Lr2 Case (a)
Lq Case (b)
Lr Case (b)
L Case (c)
La Case (d)
Lu Case (d)
Critical line
growth
normalised stretching rates
6
5
4
3
2
1
0
-1
-2
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

Fig15

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Met Office Joint Centre for Mesoscale Meteorology Department of Meteorology
University of Reading PO Box 243 Reading RG6 6BB United Kingdom
Tel: +44 (0)118 931 8425 Fax: +44 (0)118 931 8791
www.metoffice.com